

# Homework 5

(due Friday, April 14, 2023)

1. Show that for any positive numbers  $\varepsilon$  and  $A$ , we have

$$\lim_{x \rightarrow +\infty} \frac{\exp(\sqrt{\log x})}{x^\varepsilon} = 0$$

and

$$\lim_{x \rightarrow +\infty} \frac{\log^A x}{\exp(\sqrt{\log x})} = 0.$$

2. Show that

$$\sum_{n \leq x} \frac{\Lambda(n)}{n} = \log x + B + O(\exp(-c(\log x)^{1/10}))$$

for some constants  $B$  and  $c$ , with  $c > 0$ .

Hint: Use partial summation and then apply the prime number theorem with error term.

3. Show that

$$M(x) := \sum_{n \leq x} \mu(n) = O(x \exp(-c(\log x)^{1/10}))$$

for some constant  $c > 0$ .