

Lectures on Directed Graphs, Shandong  
University summer school on Graph Theory July  
6-10, 2020

Jørgen Bang-Jensen, University of Southern Denmark, Odense, Denmark

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Below BJJ refers to the online version of Bang-Jensen and Gutin: Digraphs  
first edition available at <http://www.cs.rhul.ac.uk/books/dbook/main.pdf>

## 1 July 6, 2020

- 14:00–15:30: Network flows and their applications to (di)graph problems  
Examples of topics to be covered:
  - (a) Flow decomposition
  - (b) Integrality theorem
  - (c) Max-Flow-Min-Cut Theorem
  - (d) Hoffman's circulation theorem
  - (e) Minimum value flows
  - (f) Unit capacity networks
  - (g) Path-Cycle covering number in polynomial time
  - (h) Mengers theorem.

Material BJJ Chapter 3 and Section 7.3.

- 16:00–17:30: Various topics on digraphs. Examples are
  - (a) Hamiltonian paths and cycles. BJJ Chapter 5.
  - (b) Arc-disjoint branchings (Edmonds' branching theorem). BJJ 9.5.
  - (c) (arc)-disjoint paths with prescribed ends (complexity and acyclic case). BJJ 9.2
  - (d) Spanning eulerian trails and Eulerian factors in digraphs. From Papers.
  - (e) Covering the vertices of a digraph by disjoint paths and cycles. BJJ 5.2

- (f) Submodularity of degree functions of digraphs. BJK 7.1
- (g) Strong orientations of graphs. BJK 1.6
- 19:00–20:30: Introduction to some Classes of digraphs. This includes
  - (a) tournaments and semicomplete digraphs,
  - (b) Locally semicomplete digraphs
    - In-semicomplete digraphs and Out-semicomplete digraphs.
  - (c) Extended semicomplete digraphs
  - (d) Quasi-transitive digraphs.
  - (e) Path-mergeable digraphs.

Material BJK 4.8-4.11.

## 2 July 7, 2020

- 14:00–15:30:
  - (a) Structure of locally semicomplete digraphs
  - (b) Structure of quasi-transitive digraphs. BJK 4.8-4.11.
  - (c) Examples of how to apply the structural characterizations above
- 16:00–17:30:
  - (a) Mader’s splitting theorem and Frank’s algorithm for increasing the arc-connectivity of a digraph optimally. BJK Section 7.5-7.6.
  - (b) Longest cycles in extended semicomplete digraphs and semicomplete bipartite digraphs. Material BJK 5.7-5.8.
- 19:00–20:30: Exercises:  
 BJK: 3.28(a)+(b), 3.33, 3.34, 3.35, 3.45, 3.55, 3.56 (Hint: use the integrality theorem) 3.59,3.65, 3.67, 3.70, 5.8

## 3 July 8, 2020

- 14:00–15:30: Orientations of graphs and submodular flows. Examples:
  - (a) recognizing underlying graphs of locally semicomplete digraphs and quasi-transitive graphs
  - (b) Gallai-Roy-Vitaver Theorem.
  - (c) Submodular flows
  - (d) Nash-Williams orientation theorem

Material BJG Chapter 8.

- 16:00–17:30: Disjoint directed and undirected subdigraphs in digraphs. Based on papers by Bang-Jensen and Kriesell. PDFs will be made available
- 19:00–20:30: Exercises
  - BJG 5.13, 5.14
  - BJG 7.15, 9.34 (hint: look at the proof of Lemma 7.6.2).
  - 7.11, 7.20, 7.26,7.27, 7.28, 7.30, 7.38, 7.47

## 4 July 9, 2020

- 14:00–15:30: More results on locally semicomplete digraphs and quasi-transitive digraphs. Examples are linking problems.
- 16:00–17:30: Partition problems for digraphs. Based on research papers. PDFs will be made available
- 19:00–20:30: Exercises
  - BJG 4.20, 4.31, 4.33 (hint: use Lemma 4.13 and Lemma 4.14), 4.35
  - BJG 8.1, 8.9, 8.20, 8.39, 8.46, 8.47, 8.48 (hint use the approach in Section 8.7.1 with  $D$  as the reference orientation and use flow decomposition on the associated flow which shows how to obtain  $D'$  from  $D$  by arc reversals), 8.65
  - BJG 9.59

## 5 July 10, 2020

- 14:00–15:30: Digraphs contra edge-coloured graphs. Material BJG section 11.1. Among many other things, we will show how results on cycles in bipartite digraphs are closely connected to results on cycles in 2-edge-coloured bipartite graphs.
- 16:00–17:30: Antistrong digraphs and good acyclic orientations of graphs. Based on recent works. The papers are available on ArXiv.
- 19:00–20:30: Exercises:
  - BJG 7.48
  - BJG 9.1, 9.7, 9.26, 9.27
  - More exercises will be listed later, perhaps after the course has started.

## **6 July 11, 2020**

Exam from 14:00 to 16:00 Exercises to be announced later.