短课 I

Speaker: 胡悦科 (清华大学)

Title: Subconvexity problem on higher rank groups

Abstract: In this mini-course our goal is to explain the recent approach by Paul Nelson to get subconvexity bound for L-function on higher rank groups like U_n or GL_n . We will give some numerical evidence why this approach can give nontrivial bound. For this goal we shall also review some notions and tools in the theory of automorphic forms. The main topics include

(1) Local fields, adele, idele

- (2) Basics about groups
- (3) From modular forms to automorphic forms and representations
- (4) Classification and examples of local representations
- (5) Spectral decomposition and period integrals
- (6) Relative trace formula
- (7) Application to subconvexity bound

Prerequisites: Being familiar with algebraic number theory, modular forms, basics of representation theory. I will try to review most of necessary definitions and results in the course.

References

[1] Bernstein & Zelevinsky, Induced representations of reductive *p*-adic groups I, Annales scientifiques de l'Ecole Normale Sup'erieure (1977), 441-472.

[2] D. Bump, Automorphic forms and representations, Cambridge University Press.

[3] C. Bushnell & G. Henniart, The Local Langlands Conjecture for GL₂, Springer-Verlag, Berlin, 2006.

[4] C. Bushnell & P.C. Kutzko, The admissible dual of GL(N) via compact open subgroups, volume 129 of Annals of Mathematics Studies. Princeton University Press, Princeton, NJ.

[5] P.Nelson, Spectral aspect subconvex bounds for $U_{n+1} \times U_n$, to appear in Inventiones Mathematicae. arXiv:2012.02187.

短课 Ⅱ

Speaker: 吴涵 (中国科学技术大学)

Title: Basic Introduction to Integral Representation of Automorphic L-Functions

Abstract: In this course we will present the fundamental theories on the integral representations of automorphic L-functions for GL(1) and GL(2), namely Tate's thesis, Godement-Jacquet zeta integrals and Rankin-Selberg theory for GL(2) × GL(1). The emphasis will be put on the connection with the classical theory of Dirichlet L-functions and modular forms, so that the material should be accessible to an audience with knowledge only in classical theories. A clarification on the commonly abused terminology in the literature "spectral decomposition" will also be given. As an application at the end, we will present a distributional version of Motohashi's formula, which relates the fourth moment of GL(1) L-functions with the cubic moment of GL(2) L-functions.

学术报告

Speaker: Lucile Devin (Centre Universitaire de la Mi Voix)

Title: Extending the unconditional support in an Iwaniec--Luo--Sarnak family

Abstract: We study the harmonically weighted one-level density of low-lying zeros of L-functions attached to holomorphic cusps forms of fixed even weight k and prime level tending to infinity. This family was proved to be of orthogonal type by Iwaniec, Luo and Sarnak who obtained the predicted main term for test functions having Fourier transform supported in (-3/2,3/2) unconditionally. Using zero-density estimates for Dirichlet L-function, we extend this admissible support to $(-\Theta_k, \Theta_k)$, where $\Theta_2 = 1.866 \dots$ and $\Theta_k \rightarrow 2$ as k grows.

This is joint work with Daniel Fiorilli and Anders Södergren.

Speaker: 刘旭金 (香港大学)

Title: The first sign change of the Fourier coefficients of modular forms

Abstract: In this talk, we consider the problem of the location of the first sign change among the Fourier coefficients of a modular form when all the Fourier coefficients are real. The problem has been studied by Iwaniec, Kohnen and Sengupta, Kowalski, Lau, Soundararajan and Wu, as well as Matomaki when the modular form is a primitive cusp form. For general cusp forms, there are also important works due to Siegel, Choie and Kohnen, He and Zhao, etc.. We shall give an exposition of this problem, especially on the latter case.

Speaker: 缪欣晨 (University of Minnesota, Twin Cities)

Title: Local Integrability of Bessel functions on GL(n)

Abstract: The study of Bessel functions plays an important role in number theory, automorphic forms and Langlands program. In my talk, we will focus on the Bessel functions over non-archimedean local fields. I will report on my recent work which proves that the Bessel function is locally integrable on $GL_n(\mathbb{Q}_p)$ for all $n \ge 2$, where \mathbb{Q}_p is a non-archimedean local field. The proof involves various tools in number theory and representation theory.

Speaker: 苏峰 (西交利物浦大学)

Title: Rankin-Selberg integral and its Sobolev norm

Abstract: Given any local field k and a principal series representation of G = GL(n, k), we introduce a natural integral on some open dense subset of the flag variety $B \setminus G$ (B is a Borel) and study its analytic properties. We show that such integral is proportional to the Rankin-Selberg integral with explicit multiple scalar. We also study certain Sobolev norm of the Rankin-Selberg integral in Archimedean cases. Joint work with Dongwen Liu and Binyong Sun.

Speaker: 王梦迪 (KTH Royal Institute of Technology)

Title: Counting polynomial configurations in sparse sets

Abstract: Suppose that $x + P_1(y), \dots, x + P_k(y)$ are polynomials of integer coefficients. In joint work with Lilian Matthiesen, we establish the asymptotic of counting the above polynomial sequence in primes if the polynomial sequence is of finite complexity. This generalizes and strengthens a result of Tao and Ziegler.

Speaker: 易少云 (厦门大学)

Title: Siegel modular forms of degree 2 and representation theory of GSp(4)

Abstract: In this talk, we will report some recent results on Siegel modular forms of degree 2 and representation theory of GSp(4). In particular, we will describe how to use local and global representation theory of the algebraic group GSp(4) to obtain new dimension formulas of certain family of Siegel modular forms of degree 2. We will also discuss some applications of this kind of dimensional data to other related topics in number theory. This is partially joint work with Manami Roy and Ralf Schmidt.

Speaker: 张翀 (南京大学)

Title: Local theta correspondence and local periods

Abstract: Theta correspondence can be used to study the relation between certain periods of the representations of the reductive dual pairs. This approach has been vastly studied by many mathematicians, especially by Dipendra Prasad, Wee Teck Gan, their collaborators and students. I will report some recent advances on this topic, focusing on the case of Galois periods.