Monetary Theory and Policy

Chapter 4： The Meaning of Interest Rates

Present Value

- A dollar paid to you one year from now is less valuable than a dollar paid to you today
- Why?
  - A dollar deposited today can earn interest and become $\$ 1 \times (1 + i)$ one year from today.

Discounting the Future

- Let $i = .10$

In one year

$$\$100 \times (1+0.10) = \$110$$

In two years

$$\$110 \times (1+0.10) = \$121 \text{ or } 100 \times (1+0.10)^2$$

In three years

$$\$121 \times (1+0.10) = \$133 \text{ or } 100 \times (1+0.10)^3$$

In *n* years

$$\$100 \times (1+i)^n$$

Simple Present Value

PV = present value

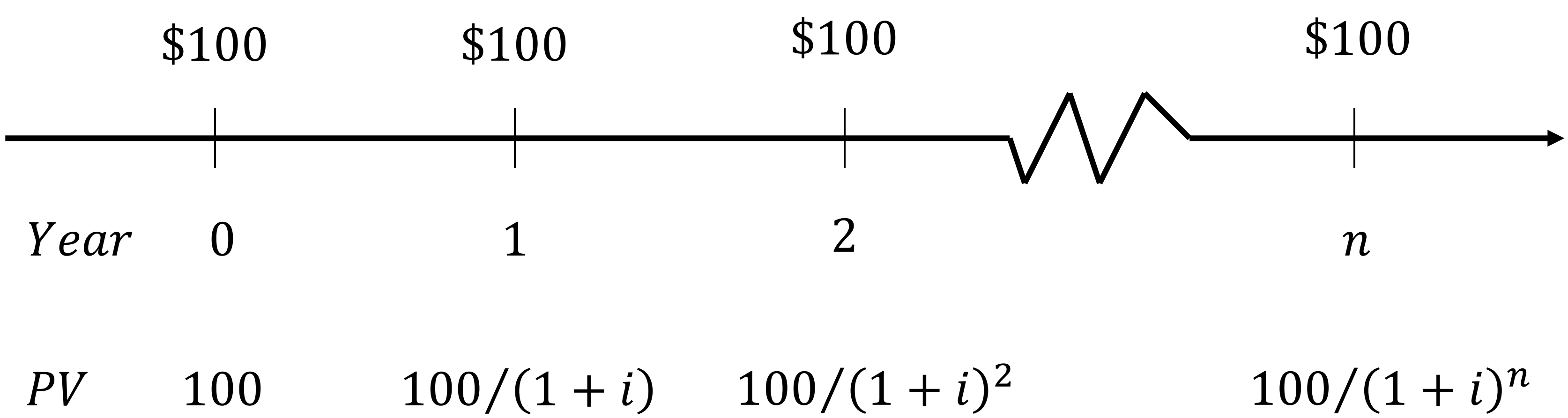
FV = future value

i = the interest rate

$$PV = \frac{FV}{(1 + i)^n}$$

Time Line

- Cannot directly compare payments scheduled in different points in the time line



Four Types of Credit Market Instruments

- Simple Loan
- Fixed Payment Loan
- Coupon Bond
- Discount Bond (zero-coupon bond)

Yield to Maturity

- The interest rate that equates the present value of cash flow payments received from a debt instrument with its value today

Simple Loan

- lender provides borrower a certain amount of funds which has to be repaid to the lender at maturity date along an additional payment for the interest

PV = amount borrowed

FV = cash flow in one year = principal+interest

n = number of years

$$PV = \frac{FV}{(1+i)^n}$$

Simple Loan

- PV = amount borrowed = \$100

  CF = cash flow in one year = \$110

  n = number of years = 1
- Then

$$\$100 = \frac{\$110}{(1+i)^1} \quad (1+i)\,\$100 = \$110$$

$$(1+i) = \frac{\$110}{\$100} \quad i = 0.10 = 10\%$$

- For simple loans, the simple interest rate equals the yield to maturity

Fixed Payment Loan

- The same cash flow payment every period throughout the life of the loan

LV = loan value

FP = fixed yearly payment

n = number of years until maturity

$$LV = \frac{FP}{1+i} + \frac{FP}{(1+i)^2} + \frac{FP}{(1+i)^3} + \ldots + \frac{FP}{(1+i)^n}$$

Fixed Payment Loan

- Mortgage: housing or consumption of durable goods

$$\text{LV} = \text{loan value of a house} = \$3{,}000{,}000$$
$$\text{FP} = \text{fixed annual payment} = \$150{,}000$$
$$n = \text{number of years until maturity} = 25$$
$$\$3{,}000{,}000 = \frac{\$150{,}000}{1+i} + \frac{\$150{,}000}{(1+i)^2} + \frac{\$150{,}000}{(1+i)^3} + \ldots + \frac{\$150{,}000}{(1+i)^{25}}$$

Coupon Bond

- Using the same strategy used for the fixed-payment loan

P = price of coupon bond

C = yearly coupon payment

F = face value of the bond

n = years to maturity date

$$P = \frac{C}{1+i} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \ldots + \frac{C}{(1+i)^n} + \frac{F}{(1+i)^n}$$

Table 1 Yields to Maturity on a 10%-Coupon-Rate Bond Maturing in Ten Years (Face Value = $1,000)

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Price of Bond ($)	Yield to Maturity (%)
1,200	7.13
1,100	8.48
1,000	10.00
900	11.75
800	13.81

- When the coupon bond is priced at its face value, the yield to maturity equals the coupon rate
- The price of a coupon bond and the yield to maturity are negatively related
- The yield to maturity is greater than the coupon rate when the bond price is below its face value

Consol or Perpetuity

- A bond with no maturity date that does not repay principal but pays fixed coupon payments forever

$$P = \frac{C}{1+i_c} + \frac{C}{(1+i_c)^2} + \frac{C}{(1+i_c)^3} + \ldots = \frac{C}{1+i_c} / \left(1 - \frac{1}{1+i_c}\right)$$

$$P = C / i_c$$

$$P_c = \text{price of the consol}$$

$$C = \text{yearly interest payment}$$

$$i_c = \text{yield to maturity of the consol}$$

$$i_c = C / P_c$$

- For coupon bonds, this equation gives the current yield, an easy to calculate approximation to the yield to maturity

Discount Bond

- Zero-Coupon Bond
- Bought at a price below its face value and the face value repaid at the maturity date.
- For any on year discount bond

F = Face value of the discount bond

P = current price of the discount bond

$$P = \frac{F}{1+i}$$

$$i = \frac{F - P}{P}$$

Rate of Return

- The payments to the owner plus the change in value expressed as a fraction of the purchase price

$$\text{RET} = \frac{C}{P_t} + \frac{P_{t+1} - P_t}{P_t}$$

- Note

$$\text{RET} = \text{return from holding the bond from time } t \text{ to time } t+1$$

$$P_t = \text{price of bond at time } t$$

$$P_{t+1} = \text{price of the bond at time } t+1$$

$$C = \text{coupon payment}$$

$$\frac{C}{P_t} = \text{current yield} = i_c$$

$$\frac{P_{t+1} - P_t}{P_t} = \text{rate of capital gain} = g$$

Rate of Return and Interest Rates

- The return equals the yield to maturity only if the holding period equals the time to maturity
- A rise in interest rates is associated with a fall in bond prices, resulting in a capital loss if time to maturity is longer than the holding period
- The more distant a bond's maturity, the greater the size of the percentage price change associated with an interest-rate change

Rate of Return and Interest Rates (cont'd)

- The more distant a bond's maturity, the lower the rate of return occurs as a result of an increase in the interest rate
- Even if a bond has a substantial initial interest rate, its return can be negative if interest rates rise

Table 2 One-Year Returns on Different-Maturity 10%-Coupon-Rate Bonds When Interest Rates Rise from 10% to 20%

(1) Years to Maturity When Bond Is Purchased	(2) Initial Current Yield (%)	(3) Initial Price ($)	(4) Price Next Year* ($)	(5) Rate of Capital Gain (%)	(6) Rate of Return (2 + 5) (%)
30	10	1,000	503	−49.7	−39.7
20	10	1,000	516	−48.4	−38.4
10	10	1,000	597	−40.3	−30.3
5	10	1,000	741	−25.9	−15.9
2	10	1,000	917	−8.3	+1.7
1	10	1,000	1,000	0.0	+10.0

*Calculated with a financial calculator using Equation 3.

Interest-Rate Risk

- Prices and returns for long-term bonds are more volatile than those for shorter-term bonds
- There is no interest-rate risk for any bond whose time to maturity matches the holding period

Real and Nominal Interest Rates

- Nominal interest rate makes no allowance for inflation
- Real interest rate is adjusted for changes in price level so it more accurately reflects the cost of borrowing
- Ex ante real interest rate is adjusted for expected changes in the price level
- Ex post real interest rate is adjusted for actual changes in the price level

Fisher Equation

- Fisher equation

$$i = i_r + \pi^e$$

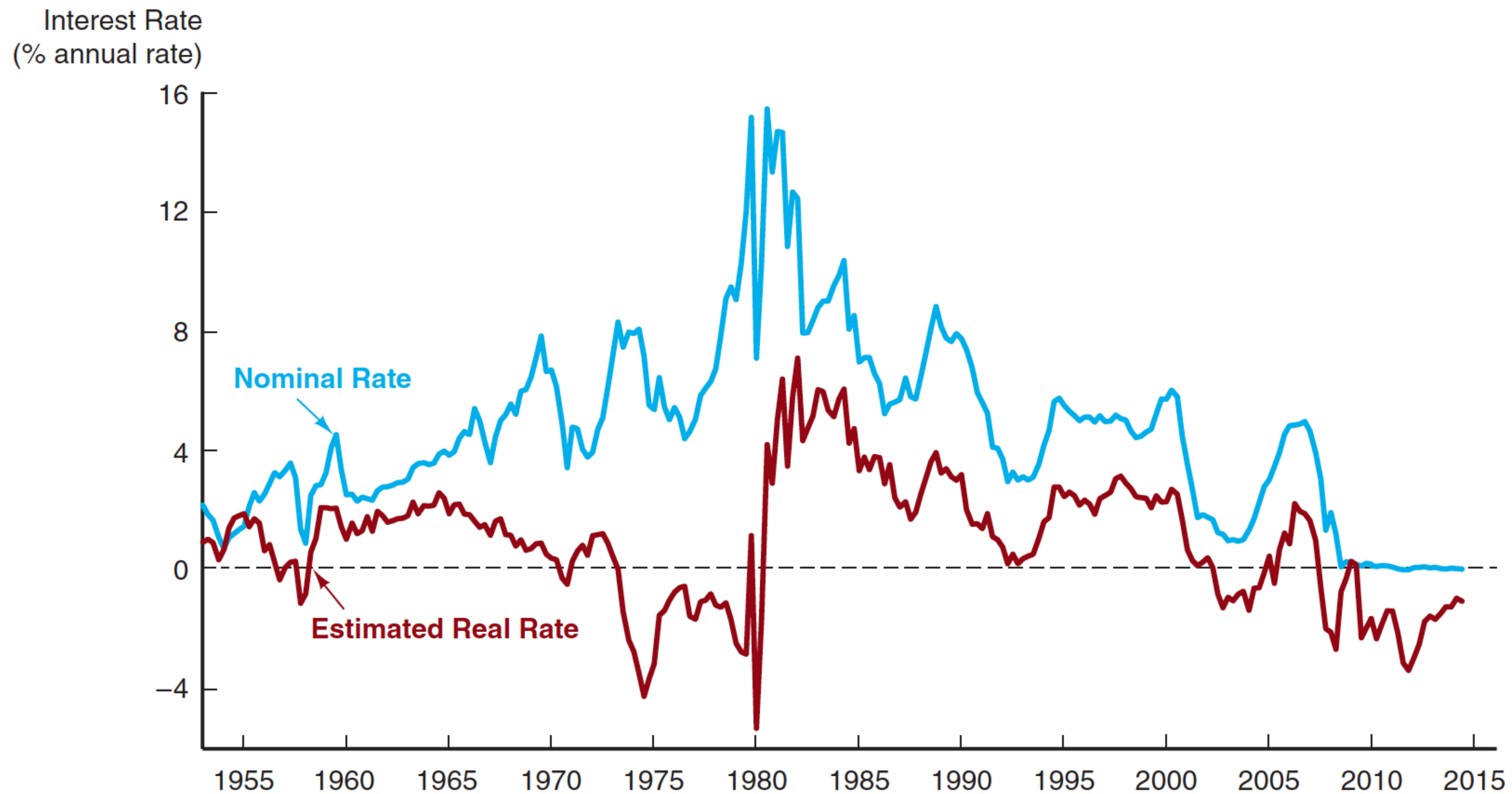
i = nominal interest rate

i_r = real interest rate

π^e = expected inflation rate

- When the real interest rate is low, there are greater incentives to borrow and fewer incentives to lend.
- The real interest rate is a better indicator of the incentives to borrow and lend

FIGURE 1 Real and Nominal Interest Rates (Three-Month Treasury Bill), 1953–2014



Sources: Nominal rates from Federal Reserve Bank of St. Louis FRED database: http://research.stlouisfed.org/fred2/. The real rate is constructed using the procedure outlined in Frederic S. Mishkin, "The Real Interest Rate: An Empirical Investigation," Carnegie-Rochester Conference Series on Public Policy 15 (1981): 151–200. This procedure involves estimating expected inflation as a function of past interest rates, inflation, and time trends, and then subtracting the expected inflation measure from the nominal interest rate.